# Review on Use of End-Area-Rule for Volume Calculations in Highway Pavements

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## ABSTRACT

Volume estimation of highway pavements is very important in planning and design of highways, in order to calculate the amount of materials and hence for cost analysis. It is also used to investigate the workmanship and quality of construction in post analysis. End area rule is the simplest and most widely used technique in volume calculation as it can be used with the cross sections generated at a suitable interval. The theory being used for road segment between two consecutive cross sections assuming there exists a linear variation of cross sectional areas within the interval. Validity of the application of end area rule depends on this assumption and accuracy also varies when the linear variation does not exist. This study was carried out to analyze the validity of the application of end area rule due to the non-linearity of variation of cross sectional areas in highway pavements. The scope of the study was limited to straight segment with rectangular cross section. Road width and height of the cross section were selected as variables and the volumes were compared using different models with coaxial centroids and skew sections. Numerical models with different angles of centroids were used in the study and the results were verified using digital models created with AutoCAD 3D software. The interval between sections was also investigated and it was determined that the perpendicular distance between sections should be used in the end area rule. Finally it was concluded that the area variation is not linear when there are more than one variable affect the cross section. The modification to end area rule was also developed in order to increase the accuracy.

**KEYWORDS:** *end-area-rule, non-linearity, volume estimation, sectional interval*

## 1 INTRODUCTION

The volume calculation is a very important event in the design stage of an engineering project as the sustainability begins with proper design. Accuracy of the volume calculation effects the estimation of material volume, workmanship and finally the total project cost. Estimation of the earth work volume of road construction project is the most crucial factor on which the highest weight of the total cost depends on. It is a challenging issue to calculate the exact volume since the amount of field measurements increases as the variation of the terrain becomes rugged. There is no any method to calculate this volume in a perfect way, but many approaches are used to approximate the result.

The accuracy of the final volume depends on the compatibility of the basis of the method and the field conditions. The most widely used method to calculate the volumes in road construction is the use of cross sections. It advances other methods by ease, time consumption and also the economy. The ability to conduct the method using contour maps is also an additional advantage. The same technique is used to calculate the volumes in reservoirs by using area enclosed by depth contours instead of cross sectional areas. There are several equations used to calculate the volume between two consecutive cross sections and end-area-rule is the most widely used method among them. According to the end-area-rule, the volume is calculated as the multiplication of the average sectional area and the perpendicular distance between the sections. The two major assumptions in the theory are the parallelism of the cross sections and the linear variation of the sectional areas which is considered as the basis of the equation. Both of the above conditions must be sacrificed to obtain a closer result but the method is applied in road constructions without paying adequate consideration for these criterions. It is obvious that the use of parallel cross section is not possible while taking the cross sections along a selected center line and the factor is deliberately skipped in the calculation stage.

This study was conducted to analyze the effect of linearity between the two cross sections when the end-area-rule is used for volume calculations. The variation of cross sectional areas was categorized as one dimensional and two dimensional according to the number of directions they are varying. Initially, rectangular, parallel sections were selected and the variation of centroid parallel to the edge of the section is considered as one dimensional variation. A numerical model was developed using basic mathematical operations and the results were confirmed using digital model generated using computer software. The above results deployed that the volume calculated by the direct application of end-area-rule deviates from the actual volume and a correlation factor can be generated to modify the final result.

## 2 LITERATURE REVIEW

It is a basic requirement to estimate the area and volumes in most of the engineering schemes such as route, alignment, reservoirs, construction of tunnels. The most significant and costly aspect of such schemes are excavation and hauling of material on which the majority of project cost is depend on. Many researches have been conducted to develop a method to calculate volumes perfectly but no perfect solution is derived. The closure approximations can be achieved by careful investigations on the field conditions as well as the theory used for the calculation.

Prismoidal method (Hickerson 1964) is one of the very basic methods used to calculate earthwork volumes in very early stage which considers the average end area concept. This method was conventionally used and became the most widely used method among these as it requires simple linear measurements. But the main disadvantage of the method is the assumption of linear variation between the sections. The average end area methods require cross-section areas to be of the same type as either cut or fill.

End area rule for cross sections with different configurations was developed by Epps and Corey (1990) to estimate earthwork volumes differently. Cheng (2005) introduced a mechanism to solve the inaccuracy problem caused by average end-area method and prismoidal method used for the calculation of roadway earthwork volume. The feasibility of average-end-area method for earthwork volume was reconfirmed by Cheng and Jiang (2013) and the difference of accuracy between 3D method and average-end-area method was analysed. It depicted that the critical value of interval distance between two consecutive cross sections is 30m for average-end-area method.

According to Khalil (2015), the average end area method is identified as tedious and time consuming technique. This literature study was intensively conducted on many models for accurately estimating earthwork volumes. The average end area model and prismoidal model were commonly employed for estimating earthwork volumes. The prismoidal model gave an exact volume for linear profiles, while the average end area model generally overestimated the volume. Cheng and Jiang (2013) further stated that a reliable and accurate earthwork volume calculation is one of the most important components in roadway engineering that can influence the choosing of roadway alignment, the cost and construction.

Roadway design has stepped into 3D era as the appearance and wide use of application of Digital Terrain Model (DTM) and accordingly 3D method for earthwork volume calculation is also developed. But the concept of adopting average-end-area method which is considered as 2D method is deep-rooted in roadway design. It has been further investigated the accuracy comparison of Roadway Earthwork Computation between 3D and 2D Methods. Weighted ground elevation was introduced by Goktepe et.al (2009) which is a method that considered the material properties in grade line selection to balance the cut and fill volume. All these researches were conducted by average-end-area method.

Easa (1992 ) introduced improvements in average-end-method when the studies on imprecision and limitations of 2D method in volume calculation are considered. Aruga and Akay (2005) developed a forest road design program based on a high-resolution Digital Elevation Model (DEM) from a light detection and ranging (LIDAR) system. After a designer had located the intersection points on a horizontal plane, the model firstly generated the horizontal alignment and the ground profile, and then it could precisely generate cross-sections and accurately calculate earthwork volumes using a high-resolution DEM. A shortage of this model was the incapability of properly optimizing horizontal and vertical alignments simultaneously. The researchers further used DTM to calculate cross section area, but still completed volume computation by 2D rule. These programs had begun to bring DTM into roadway design and volume calculation, but actually it can not be considered as pure 3D concept as they still use average-end-area or prismoidal method to compute earthwork volume finally.

Even though this rule is developed for the sections with parallel sections, its applicability for the curved road sections have been investigated (Wanasinghe et.al, 2018) and concluded that the end-area-rule can be used with modifications applied according to the skew angle. All these studies prove that this rule is a valid tool to calculate the volumes in construction industry despite that the ideal conditions are not satisfied.

## 3 METHODOLOGY

This study was carried out to investigate the factor of linearity of the variation of cross sectional area and the scope of the study was limited to parallel rectangular sections. The edges of the rectangles were kept parallel and the width of each section was kept constant while only the lengths are different which is called the one dimensional variation. The situation where the both length and width of the two surfaces are different is termed as two dimensional variation.

Two models called numerical model and digital model were generated to calculate the volumes for different dimensions and to confirm the calculated volume. The calculated volumes were compared with the conventionally calculated volume by end area rule.

### 3.1 Numerical Model

The volume of the pavement section was calculated by developing a mathematical equation using integration. Two parallel rectangular sections having dimensions ***c*** and ***d*** at the top and ***a*** and ***b*** at the bottom which are distance ***H*** apart is considered as shown in Figure 1. The one dimensional variation was obtained by considering the situation where the dimensions ***a*** and ***c*** become identical.

It is proved with the fundamentals that the linearity of the variation of end areas exists in this situation and the volume of the section become identical with the end-area-rule which is given in equation (1)

$$V = h(ab + cd)/2 \tag{1}$$

The existence of the above condition is very rare in the practical applications and the two dimensionally varied modules were selected with the same dimensions but no edges in same vertical plane. The equation derived using basic mathematical integration is given in equation (2).

$$V = h(2ab + 2cd + ad + bc)/6 \tag{2}$$

### 3.2 Digital Model

The volumes of the sections used in numerical models were determined by generating the similar models using computer software Auto Cad 2019 which allows measuring the volume directly. Figure 2 illustrates a digital model used to measure the volume using Auto Cad 2019.

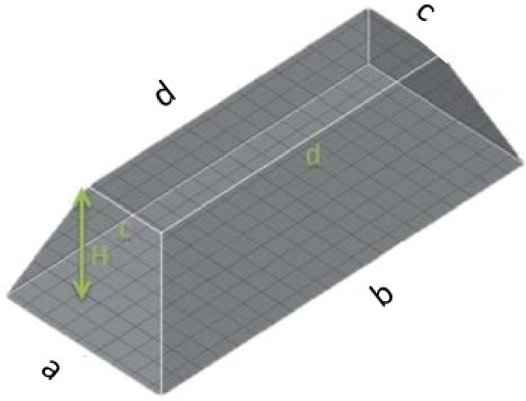


Figure 1. Details of the numerical model

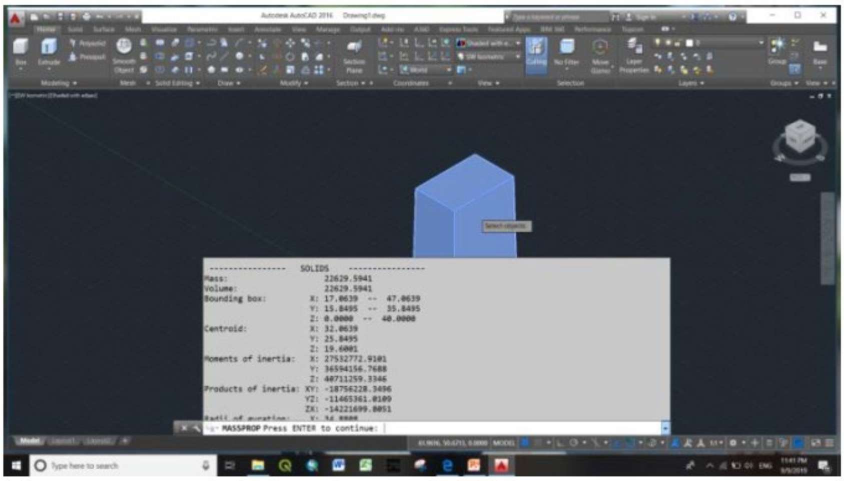

Figure 2. Digital Model and its details

## 4 OBSERVATION AND RESULTS

12 samples were used in this investigation whose dimensions are depicted in Table 1.

Table 1. Dimensions of the samples

| Trial No. | Dimensions (m) | | | | |
|---|---|---|---|---|---|
| | a | b | c | d | h |
| 1 | 30 | 20 | 44.1062 | 34.1062 | 40 |
| 2 | 30 | 20 | 36.9991 | 26.9991 | 40 |
| 3 | 30 | 20 | 35.5941 | 25.5941 | 40 |
| 4 | 30 | 20 | 34.1926 | 24.1926 | 40 |
| 5 | 30 | 20 | 32.6937 | 22.6937 | 40 |
| 6 | 30 | 20 | 31.3964 | 21.3964 | 40 |
| 7 | 30 | 20 | 28.6036 | 18.6036 | 40 |
| 8 | 30 | 20 | 27.3063 | 17.3063 | 40 |
| 9 | 30 | 20 | 25.8074 | 15.8074 | 40 |
| 10 | 30 | 20 | 24.4059 | 14.4059 | 40 |
| 11 | 30 | 20 | 23.0009 | 13.0009 | 40 |
| 12 | 30 | 20 | 15.8938 | 5.8938 | 40 |

Volumes of each sample were calculated using traditional end-area-rule, the derived formula of equation (2) and using the digital model. The values were compared with each other and the comparison is illustrated in Table 2.

Table 2. Volume calculated by different methods

| Trial No. | Volume ($m^3$) | | |
|---|---|---|---|
| | Numerical method | Digital Model | End-area-rule |
| 1 | 40759.33 | 40 759.27 | 42085.89 |
| 2 | 31652.27 | 31 652.26 | 31978.84 |
| 3 | 30011.35 | 30 011.40 | 30219.98 |
| 4 | 28426.97 | 28 427.00 | 28544.16 |
| 5 | 26790.45 | 26 897.72 | 26838.82 |
| 6 | 25422.40 | 25 422.40 | 25435.40 |
| 7 | 22629.60 | 22 629.59 | 22642.60 |
| 8 | 21403.05 | 21 310.40 | 21451.42 |
| 9 | 20041.77 | 20 041.75 | 20158.96 |
| 10 | 18823.15 | 18 823.11 | 19031.78 |
| 11 | 17654.07 | 17 654.07 | 17980.65 |
| 12 | 12546.93 | 12 546.96 | 13873.50 |

It is clearly identified that the numerical and digital models resulted very close volumes but end area rule always generated over estimations. The error analysis was conducted by considering the numerical method as the absolute value. The errors of end area rule were calculated and the corresponding values are illustrated in Table 3. To investigate the relationship between the error and the geometry of the section, the differences of the dimensions in each side was calculated and the multiplication of dimensional deviations were considered. It showed proportionality to the error as shown in the Table 3.

Table 3. Error analysis

| Trial No. | Error | a-c | b-d | (a-c)(b-d)=α | Error/ α |
|---|---|---|---|---|---|
| 1 | 1326.566 | 14.1062 | 14.1062 | 198.985 | 6.667 |
| 2 | 326.5827 | 6.9991 | 6.9991 | 48.987 | 6.667 |
| 3 | 208.6264 | 5.5941 | 5.5941 | 31.294 | 6.667 |
| 4 | 117.186 | 4.1926 | 4.1926 | 17.578 | 6.667 |
| 5 | 48.37346 | 2.6937 | 2.6937 | 7.256 | 6.667 |
| 6 | 12.99955 | 1.3964 | 1.3964 | 1.950 | 6.667 |
| 7 | 12.99955 | -1.3964 | -1.3964 | 1.950 | 6.667 |
| 8 | 48.37346 | -2.6937 | -2.6937 | 7.256 | 6.667 |
| 9 | 117.186 | -4.1926 | -4.1926 | 17.578 | 6.667 |
| 10 | 208.6264 | -5.5941 | -5.5941 | 31.294 | 6.667 |
| 11 | 326.5827 | -6.9991 | -6.9991 | 48.987 | 6.667 |
| 12 | 1326.566 | -14.1062 | -14.1062 | 198.985 | 6.667 |

## 5 CONCLUSION

From the above results, it can be concluded that the end area rule is valid only for one dimensionally varying sections. It can be verified through the error analysis since the error become zero when either ***a*** and ***c*** or ***b*** and ***d*** become identical.

The equation 2 provides better approximations than end area rule for two dimensionally varying sections. The variations in side dimensions can be used to determine a correction factor and the end-area rule can be modified to achieve more accurate volume estimations.

However for this correction factor cannot be applied for non-rectangular sections. Therefore this should be further studied and related and it is supposed to be investigated in future.

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